

Multiplicity, mean p_T , p_T -spectra and elliptic flow of identified particles in Pb+Pb collisions at LHC

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Israel-Stewart's causal theory of dissipative hydrodynamics, with the ADS/CFT lower limit of shear viscosity to entropy ratio ($\eta/s=0.08$), give consistent description of a number of experimental observables in Au+Au collisions at RHIC (c.m. energy $\sqrt{s}=200$ GeV) [1]. Assuming that in Pb+Pb collisions at LHC (c.m. energy $\sqrt{s}=5.5$ TeV), except for the initial temperature, other parameters of the fluid remain unchanged, we have predicted for the centrality dependence of multiplicity, mean p_T , p_T -spectra, elliptic flow. The central temperature of the fluid is adjusted to $T_i=421$ MeV such that in a Pb+Pb collision, with participant number $N_{part}=350$, average charge particle multiplicity is ~ 900 and is consistent with the experimental trend observed at lower energies. Compare to Au+Au collisions at RHIC, in Pb+Pb collisions at LHC, on the average, particle multiplicity increases by a factor of ~ 1.6 , the mean p_T is increased by $\sim 10\%$ only. The elliptic flow on the other hand decreases by $\sim 15\%$.

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I. INTRODUCTION

Experiments in Au+Au collisions at RHIC [2, 3, 4, 5], produced convincing evidences that in non-central Au+Au collisions, a hot, dense, strongly interacting, collective QCD matter is created. Whether the matter can be characterized as the lattice QCD [6] predicted Quark-Gluon-Plasma (QGP) or not, is still a question of debate. Relativistic hydrodynamics provides a convenient tool to analyse Au+Au collision data. It is assumed that in the collision a fireball is produced. Constituents of the fireball collide frequently to establish local thermal equilibrium sufficiently fast and after a certain time τ_i , hydrodynamics become applicable. If the macroscopic properties of the fluid e.g. energy density, pressure, velocity etc. are known at the equilibration time τ_i , the relativistic hydrodynamic equations can be solved to give the space-time evolution of the fireball till a given freeze-out condition such that interactions between the constituents are too weak to continue the evolution. Using suitable algorithm (e.g. Cooper-Frye) information at the freeze-out can be converted into particle spectra and can be directly compared with experimental data. Thus, hydrodynamics, in an indirect way, can characterize the initial condition of the medium produced in heavy ion collisions. Hydrodynamics equations are closed only with an equation of state and one can investigate the possibility of phase transition in the medium.

A host of experimental data produced in Au+Au collisions at RHIC, at c.m. energy $\sqrt{s}=200$ GeV, have been successfully analysed using ideal hydrodynamics [7]. Multiplicity, mean p_T , p_T -spectra, elliptic flow etc. of identified particles, are well explained in the ideal hydrodynamic model with QGP as the initial state. Ideal

hydrodynamics analysis of the RHIC data indicate that in central Au+Au collisions, at the equilibration time $\tau_i \approx 0.6$ fm, central energy density of the QGP fluid is $\varepsilon_i \approx 30$ GeV/fm³ [7]. It may be mentioned that ideal hydrodynamics description of data are not unblemished. p_T spectra or the elliptic flow are explained only up to transverse momenta $p_T \approx 1.5$ GeV. At higher p_T description deteriorates. Also ideal hydrodynamic description to data gets poorer in peripheral collisions.

Hydrodynamics implicitly assume local thermal equilibration. In the rest frame of the fluid, particle momenta are isotropic. But at the early stage of the evolution, assumption of isotropic momentum distribution cannot be very accurate. System could only be in partial equilibration and for a creditable analysis of the experimental data, dissipative effects must be accounted for. Shear viscosity is the most important dissipative effect in heavy ion collisions. Shear viscosity of QGP matter is quite uncertain. In recent years, string theory motivated calculations [8] indicated that shear viscosity over entropy ratio of any matter is bounded from the lower side, $\eta/s \geq 1/4\pi$. It is then expected that at the minimum, shear viscosity over entropy of QGP matter should be $1/4\pi$.

In recent years there has been significant progress in numerical implementation of viscous dynamics [1, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. At the Cyclotron Centre, Kolkata, we have developed a code "AZHYDRO-KOLKATA", to solve Israel-Stewart's 2nd order theory for dissipative hydrodynamics in 2+1 dimensions. In a recent publication [1], it was shown that minimally viscous hydrodynamics ($\eta/s=0.08$), with QGP as the initial state consistently explain a large part of RHIC data, e.g. p_T -spectra of identified particles, elliptic flow in minimum-bias/mid-central collisions etc. Indeed, the description is better than that obtained in ideal dynamics, particularly at large p_T .

In not too distant future, Large Hadron Collider (LHC)

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is expected to be operational. It is being planned to collide Lead beams at c.m. energy $\sqrt{s}=5.5$ TeV, which is 27 times larger than the RHIC energy collisions. One expects that conclusive evidence for QGP formation can be obtained in Pb+Pb collisions at LHC. In the present paper, in the framework of minimally viscous hydrodynamics, we have given predictions for several experimental observables, which will be measured in Pb+Pb collisions at LHC energy. It will be shown that even if, from RHIC to LHC collisions, energy is increased by a factor of 27, the experimental observables e.g. multiplicity, p_T spectra, mean p_T , elliptic flow etc. do show rather modest variation from RHIC energy collisions. The reason can be understood also. Multiplicity or entropy of the system increases logarithmically with energy. Thus, from RHIC to LHC, even though collision energy increases by a factor 27, the entropy or multiplicity increase by a factor of ~ 1.6 only. Experimental observables do not show very large variation from RHIC energy collisions.

centrality	PB+Pb@LHC		Au+Au@RHIC	
	$\langle N_{part} \rangle$	$\langle b \rangle$	$\langle N_{part} \rangle$	$\langle b \rangle$
0-10	346.8	3.51	318.9	3.30
10-20	245.1	6.13	223.1	5.79
20-30	169.9	7.93	154.1	7.49
30-40	113.4	9.38	102.8	8.87
40-50	71.6	10.64	65.0	10.06
50-60	41.5	11.76	37.8	11.11
60-70	21.5	12.79	19.8	12.09
70-80	9.6	13.76	9.08	13.0
80-90	3.1	14.80	2.97	14.03

TABLE I: Glauber model calculation for the average participant number $\langle N_{part} \rangle$ and average impact parameter $\langle b \rangle$ (in fm) for different ranges of centrality cuts in Pb+Pb/Au+Au collisions at LHC/RHIC energy.

II. INITIAL CONDITIONS FOR PB+PB COLLISIONS AT LHC

Details of our solution of Israel-Stewart's [20] 2nd order theory of dissipative hydrodynamics can be found in [1]. Briefly, assuming longitudinal boost-invariance, we have solved the Israel-Stewart's 2nd order theory for a baryon free fluid with dissipation due to shear viscosity only. In Israel-Stewart's theory, dissipative flows are treated as extended thermodynamic variables. For a baryon free fluid with only shear viscosity as the dissipative effect, energy-momentum conservation equation is required to be solved simultaneously with the relaxation equation for the shear stress tensor,

$$\partial_\mu T^{\mu\nu} = 0, \quad (1)$$

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - 2\eta\nabla^{<\mu}u^{\nu>}). \quad (2)$$

Eq.1 is the conservation equation for the energy-momentum tensor, $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu}$, ε , p and u being the energy density, pressure and fluid velocity respectively. Eq.2 is the relaxation equation for the shear stress tensor $\pi^{\mu\nu}$. In Eq.2, $D = u^\mu \partial_\mu$ is the convective time derivative, $\nabla^{<\mu}u^{\nu>} = \frac{1}{2}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}(\partial \cdot u)(g^{\mu\nu} - u^\mu u^\nu)$ is a symmetric traceless tensor. η is the shear viscosity and τ_π is the relaxation time. In [1] Eqs.1 and 2 are solved in $(\tau = \sqrt{t^2 - z^2}, x, y, \eta_s = \frac{1}{2} \ln \frac{t+z}{t-z})$ coordinates, assuming boost-invariance.

We note that presently there is disagreement about the form of the relaxation equation to be used in heavy ion collisions. In [15, 18] an extra term $R = [u^\mu \pi^{\nu\lambda} + u^\nu \pi^{\nu\lambda}]Du_\lambda$ is included in the relaxation equation. Shear stress tensor is traceless ($\pi^\mu_\mu = 0$) and transverse to 4-velocity ($u_\mu \pi^{\mu\nu} = 0$). The term is needed to maintain the transversality and tracelessness condition. Israel-Stewart [20] developed the theory on gradient expansion of entropy, gradients of equilibrium thermodynamical variables are assumed to be small. The term $[u^\mu \pi^{\nu\lambda} + u^\nu \pi^{\nu\lambda}]Du_\lambda$ does not contribute to entropy and is missed in Israel-Stewart's theory. Moreover, in Israel-Stewart's theory, the term can be neglected (both $\pi^{\mu\nu}$ and Du_μ are small and their combination is neglected). We have also checked that for minimally viscous fluid, contribution of the term is negligible. For minimally viscous fluid, energy density evolution is hardly affected whether the term is present or not in the relaxation equation (see Fig.8 and 9 of [1]).

Details of the analysis of RHIC data in Au+Au collisions can be found in [1]. In brief, minimally viscous QGP fluid was initialised with a Glauber model initial condition, with 75% soft collisions and 25% hard collisions. In a $b=0$ collision, this corresponds to central entropy density $S_{ini}=110 \text{ fm}^{-3}$, at the initial time $\tau_i=0.6$ fm. The transverse fluid velocity at the initial time was assumed to be zero, $v_x = v_y = 0$. The shear stress tensor $\pi^{\mu\nu}$ was assumed to attain boost-invariant value at the initial time τ_i . For the relaxation time τ_π , Boltzmann approximation $\tau_\pi = 3\eta/2p$ is used. The freeze-out temperature was varied to fit elliptic flow in 16-23% centrality Au+Au collisions. It was seen that for $T_F=130$ MeV, elliptic flow in 16-23% centrality Au+Au collisions as well as a host of other data are explained. For the equation of state we have used EOS-Q developed in [7], with bag model EOS for the QGP phase and hadronic resonance gas for the hadronic phase. EOS-Q has a 1st order phase transition at $T_c=164$ MeV.

We assume that in Pb+Pb collisions at LHC, except for the central entropy density, other parameters of the model remain unchanged. In Pb+Pb collisions also, the QGP fluid is thermalised at the same time as in Au+Au

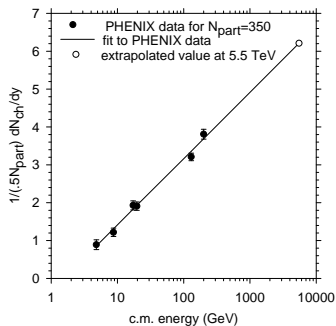


FIG. 1: Filled circles are the PHENIX data for the charged particle multiplicity per participant $\frac{1}{.5N_{part}} \frac{dN}{dy}$ as a function of c.m. energy for participant number $N_{part}=350$. The solid line is a fit to the PHENIX data by Eq.3. The unfilled circle is the extrapolated value of $\frac{1}{.5N_{part}} \frac{dN}{dy}$ at LHC energy $\sqrt{s}=5.5$ TeV, for participant number $N_{part}=350$.

collisions at RHIC i.e. $\tau_i=0.6$ fm. The initial fluid velocity is zero: $v_x = v_y = 0$, and the shear stress tensor has attained the boost-invariant value. And as in Au+Au collisions, in Pb+Pb collisions also, the hadronic fluid freezes-out at $T_F=130$ MeV. The central energy density or entropy density in Pb+Pb collisions at LHC energy ($\sqrt{s}=5.5$ TeV) cannot be same as in Au+Au collisions at RHIC ($\sqrt{s}=200$ GeV). One expects larger energy deposition in Pb+Pb collisions. To obtain the initial energy/entropy density of the fireball in LHC energy collisions we proceed as follows:

PHENIX collaboration [22] has tabulated the average charged particle multiplicity as a function of collision energy for a range of collision centrality. In Fig.1, for participant number $N_{part}=350$, the average multiplicity $\frac{1}{.5N_{part}} \frac{dN_{ch}}{d\eta}$ is shown as a function of collision energy. The multiplicity increases logarithmically with energy,

$$\frac{dN_{ch}}{d\eta} = A + B \ln \sqrt{s}, \quad (3)$$

with $A = -0.33$ and $B = 0.75$. We use the relation to extrapolate to LHC energy $\sqrt{s}=5.5$ TeV. The extrapolated value of average charged particle multiplicity in LHC energy is $\sim 927 \pm 70$. We adjust the central entropy density to $S_{ini}=180 \text{ fm}^{-3}$ such that a $N_{part}=350$ Pb+Pb collision produce ~ 900 charged particles. Entropy density $S_{ini}=180 \text{ fm}^{-3}$ corresponds to central temperature $T_i=421$ MeV. Compared to Au+Au collisions at RHIC (central temperature $T_i=357$ MeV), in Pb+Pb collisions at LHC, central temperature is $\sim 20\%$ higher.

With the initial condition as described above we have solved the hydrodynamic equations and calculate invariant particle yield from the freeze-out surface at $T_F=130$ MeV. It may be noted that we are assuming boost-invariance. Consequently, our predictions are valid only in the mid-rapidity range. In the following, we will show our predictions as a function of collision centrality or rather as a function of number of participants.

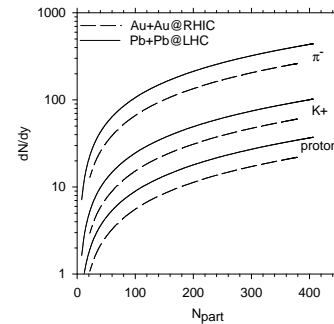


FIG. 2: The black, red and green solid and dashed lines are minimally viscous hydrodynamics predictions for the centrality dependence of π^- , K^+ and proton multiplicity in Pb+Pb collisions at LHC and in Au+Au collisions at RHIC. The yields are normalised by a factor of $N=1.4$ to account for the neglect of resonance production.

Glauber model calculations for average participant number $\langle N_{part} \rangle$ and average impact parameter $\langle b \rangle$ for different centrality cuts are given in table I. For Pb+Pb collisions at LHC energy, we have used $\sigma_{inel}=70$ mb. For comparison, in table I, we have also shown the same results for Au+Au collisions at RHIC, when $\sigma_{inel}=44$ mb.

III. MULTIPLICITY AND MEAN p_T IN PB+PB COLLISIONS AT LHC

Particle multiplicity is one of the important observables in heavy ion collisions. It is a measure of the entropy of the system. In Fig.2, the black, red and green lines are minimally viscous hydrodynamic model predictions for the centrality dependence of π^- , K^+ and proton multiplicity in Pb+Pb collisions at LHC. For comparison, minimally viscous hydrodynamic predictions for π^- , K^+ and proton multiplicity in Au+Au collisions at RHIC are also shown (the dashed lines) in Fig.2. We have neglected resonance production. To account for the neglect of resonance production, yields are normalised by a factor of $N = 1.4$. From the predictions, it appears that compared to Au+Au collisions, in central/mid-central Pb+Pb collisions, particle yields are enhanced by a factor of $\sim 1.6-1.8$. It is expected. Multiplicity increases logarithmically with energy.

Centrality dependence of mean p_T is another important observable. In Au+Au collisions at RHIC, in minimally viscous hydrodynamics, centrality dependence of $\langle p_T \rangle$ of identified particles are reasonably well explained. In Fig.3 minimally hydrodynamics predictions for mean p_T for π^- , K^+ and protons, in Pb+Pb collisions at LHC are shown. In collisions beyond $N_{part}=50$, mean $\langle p_T \rangle$ is approximately constant; $\langle p_T \rangle \approx 0.5, 0.75$ and 1 for π^- , K^+ and protons respectively. For comparison, predictions for mean p_T in Au+Au collisions at RHIC are shown in Fig.3 as the dashed lines. From RHIC to LHC, even though collision energy is increased by a

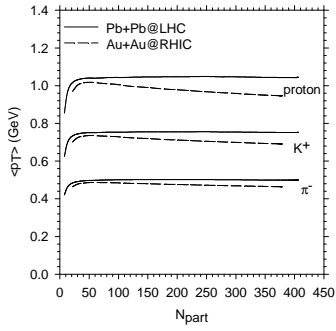


FIG. 3: The solid lines are predicted mean p_T for π^- , K^+ and proton in Pb+Pb collisions at LHC energy. For comparison, minimally viscous hydrodynamics predictions for mean p_T in Au+Au collisions are also shown by the dashed lines.

factor of 27, the mean p_T is increased marginally. For example, in a central collision with $N_{part}=350$, for all the species, mean p_T is increased by $\sim 10\%$ from RHIC to LHC energy.

IV. p_T -SPECTRA IN PB+PB COLLISIONS AT LHC

Minimally viscous hydrodynamic model predictions for π^- p_T -spectra, in 0-10%, 10-20%, 20-30%, 30-40%, 40-50% and 50-60% centrality Pb+Pb collisions at LHC are shown in Fig.4. We have neglected resonance production. Resonance decay contribute to particle yield, more at low p_T than at large p_T . For example, at freeze-out temperature $T_F=150$ MeV, nearly $\sim 50\%$ of total pions are from resonance decay at $M_T=0.5$ GeV, contribution of decay pions decreases to $\sim 20\%$ at higher $M_T=2$ GeV [23]. Minimally viscous hydrodynamics predictions, normalised by a factor of $N=1.4$, well explained the π^- p_T spectra in Au+Au collisions. However, we must mention that overall normalisation do not account correctly for the resonance contribution to particle p_T -spectra. p_T spectra will be uncertain by 10-15%. For comparison, in Fig.4, predicted spectra in Au+Au collisions at RHIC are shown as the dashed lines. p_T spectra are slightly flattened at LHC. It is consistent with the predicted small increase in mean p_T (see Fig.3) in LHC energy.

In Fig.5 and 6, we have shown the predicted p_T spectra for K^+ and protons. We have not shown, but here again, compared to RHIC energy p_T spectra is flattened. Before we digress, we would like to note that even though we have shown predictions right up to $p_T=5$ GeV, at large $p_T > 3$ GeV, there may be other sources (e.g. jets) for particle production. It is unlikely that viscous dynamics will predict correctly particle production at high $p_T > 3$ GeV. Present predictions are expected to be in reasonable agreement with future experiments up to $p_T \sim 3$ GeV.

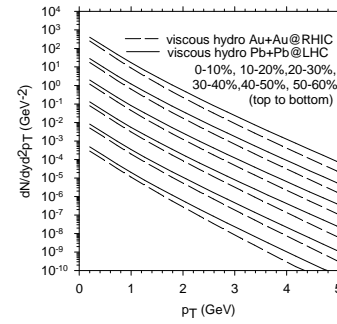


FIG. 4: The solid lines are minimally viscous hydrodynamic predictions for π^- p_T spectra in 0-10%, 10-20%, 20-30%, 30-40%, 40-50% and 50-60% centrality Pb+Pb collisions at LHC. The dashed lines are the same for Au+Au collisions at RHIC. To account for neglect of resonance production, the yield are normalised by a factor of 1.4.

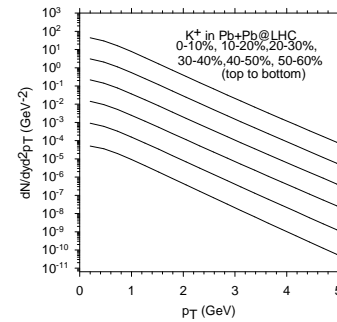


FIG. 5: The solid lines are minimally viscous hydrodynamics predictions for the K^+ invariant yield in Pb+Pb collisions at LHC energy.

V. ELLIPTIC FLOW IN PB+PB COLLISIONS AT LHC

One of the important observations in Au+Au collisions at RHIC is the significant elliptic flow in non-central collisions. Qualitatively, elliptic flow is explained in a hydrodynamic model, re-scattering of secondaries generates pressure and drives the subsequent collective motion. In non-central collisions, the reaction zone is asymmetric (almond shaped), pressure gradient is large in one direction and small in the other. The asymmetric pressure gradients generate the elliptic flow. As the fluid evolve and expands, asymmetry in the reaction zone decreases and comes a stage when the reaction zone become symmetric and system no longer generate elliptic flow. Elliptic flow is an early time phenomena and a sensitive probe to the early stage of the fluid.

In Fig.7, solid line is the predicted elliptic flow in minimum bias Pb+Pb collisions at LHC. For comparison, we have shown the minimum bias elliptic flow in Au+Au collisions at RHIC (the dashed line). It is not shown here, but experimental minimum bias v_2 in Au+Au collisions is well reproduced in minimally viscous hydrodynamics. It is interesting to note that in Pb+Pb collisions, elliptic

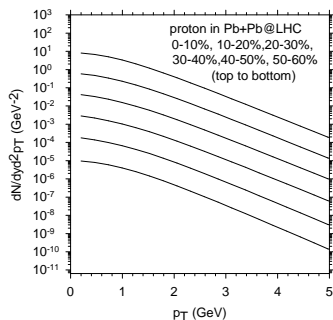


FIG. 6: same as in Fig.5 but for protons.

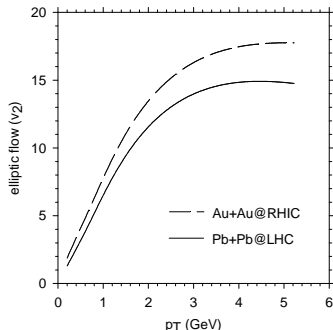


FIG. 7: The solid line is the minimally viscous hydrodynamics prediction for the minimum bias elliptic flow in Pb+Pb collisions at LHC. For comparison, minimum bias elliptic flow in Au+Au collisions at RHIC is also shown (the dashed line).

flow is reduced by $\sim 15\%$. Reduction of elliptic flow in LHC has also been predicted by Krieg and Bleicher [25]. In a parton recombination model, they studied the energy dependence of elliptic flow in heavy ion collisions from AGS to LHC energy. It was observed that from RHIC to LHC energy elliptic flow decreases. Parton transport models also predict less elliptic flow in LHC energy than in Au+Au collisions at RHIC [26].

In Fig.8, in four panels, we have shown the minimally viscous dynamics predictions for elliptic flow in 0-10%, 10-20%, 20-30% and, 30-40% centrality Pb+Pb collisions at LHC. The black, red and green lines corresponds to elliptic flow for π^- , K^+ and protons. Species dependence is similar to that at RHIC energy. At large p_T , elliptic flow between different species are marginally different. Species dependence is seen only at low p_T , lighter the particle, more is the elliptic flow. We have not shown any comparison with v_2 at RHIC. But as with the minimum

bias collisions, elliptic flow in different centrality ranges of collisions is reduced at LHC.

VI. SUMMARY

To summarise, in a minimally viscous ($\eta/s=0.08$) hydrodynamics, we have given predictions for several experimental observables in Pb+Pb collisions at LHC en-

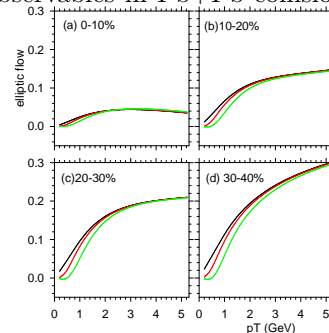


FIG. 8: (color online) Predicted elliptic flow in 0-10%, 10-20%, 20-30% and 30-40% Pb+Pb collisions at LHC are shown. The black, red and green lines are elliptic flow for π^- , K^+ and protons respectively.

ergy ($\sqrt{s}=5.5$ TeV). Assuming that particle multiplicity increase logarithmically with c.m. energy, we have extrapolated the lower energy data (tabulated by the PHENIX collaboration) to obtain an estimate of particle multiplicity at LHC energy collisions. Our estimate indicate that a Pb+Pb collision with participant number $N_{part}=350$, produces ~ 927 charged particles. The initial central entropy density of the fluid ($S_{ini}=180 \text{ fm}^{-3}$) in Pb+Pb collisions was fixed to reproduce the extrapolated particle multiplicity. The initial time (τ_i) and freeze-out temperature (T_F) were kept fixed at the value, $\tau_i=0.6$ fm and $T_F=130$ MeV, as it was obtained in the analysis of Au+Au data in RHIC energy collisions.

Our predictions indicate that compared to Au+Au collisions at RHIC, in Pb+Pb collisions at LHC, (i) particle multiplicity will increase by a factor of ~ 1.6 , (ii) the mean p_T will be enhanced by $\sim 10\%$, (iii) p_T spectra of identified particles will be (slightly) flattened and (iv) elliptic flow will decrease by $\sim 15\%$. We hope the results will be helpful in planning future experiments in LHC energy.

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- [1] A. K. Chaudhuri, arXiv:0801.3180 [nucl-th].
 - [2] BRAHMS Collaboration, I. Arsene *et al.*, Nucl. Phys. A **757**, 1 (2005).
 - [3] PHOBOS Collaboration, B. B. Back *et al.*, Nucl. Phys. A **757**, 28 (2005).
 - [4] PHENIX Collaboration, K. Adcox *et al.*, Nucl. Phys. A

- 757** 184 (2005).
- [5] STAR Collaboration, J. Adams *et al.*, Nucl. Phys. A **757** 102 (2005).
- [6] Karsch F, Laermann E, Petreczky P, Stickan S and Wetzorke I, 2001 *Proceedings of NIC Symposium* (Ed. H. Rollnik and D. Wolf, John von Neumann Institute for

- Computing, Jülich, NIC Series, vol.9, ISBN 3-00-009055-X, pp.173-82,2002.)
- [7] P. F. Kolb and U. Heinz, in *Quark-Gluon Plasma 3*, edited by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004), p. 634.
 - [8] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001).
 - [9] D. Teaney, Phys. Rev. C **68**, 034913 (2003) [arXiv:nucl-th/0301099].
 - [10] A. Muronga and D. H. Rischke, nucl-th/0407114 (v2).
 - [11] T. Koide, G. S. Denicol, Ph. Mota and T. Kodama, Phys. Rev. C **75**, 034909 (2007).
 - [12] A. K. Chaudhuri and U. W. Heinz, J. Phys. Conf. Ser. **50**, 251 (2006).
 - [13] U. W. Heinz, H. Song and A. K. Chaudhuri, Phys. Rev. C **73**, 034904 (2006).
 - [14] A. K. Chaudhuri, Phys. Rev. C **74**, 044904 (2006). arXiv:nucl-th/0703027; arXiv:nucl-th/0703029; arXiv:0704.0134 [nucl-th].
 - [15] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007) [arXiv:0706.1522 [nucl-th]].
 - [16] P. Romatschke, Eur. Phys. J. C **52**, 203 (2007) [arXiv:nucl-th/0701032].
 - [17] R. Baier and P. Romatschke, Eur. Phys. J. C **51**, 677 (2007) [arXiv:nucl-th/0610108].
 - [18] H. Song and U. W. Heinz, arXiv:0709.0742 [nucl-th].
 - [19] H. Song and U. W. Heinz, arXiv:0712.3715 [nucl-th].
 - [20] W. Israel, Ann. Phys. (N.Y.) **100**, 310 (1976); W. Israel and J. M. Stewart, Ann. Phys. (N.Y.) **118**, 349 (1979).
 - [21] It has been argued [18] that during the evolution, the term $[u^\mu \pi^{\nu\lambda} + u^\nu \pi^{\mu\lambda}] Du_\lambda$, is needed to maintain the transversality of $\pi^{\mu\nu}$ with u^μ . The argument is incorrect. Multiplying Eq.2 by u_μ , one can check that if $\pi^{\mu\nu}$ and Du^μ are small and the combination $\pi^{\mu\nu} Du_\mu$ can be neglected, the transversality condition is maintained. Indeed, it was pointed out in [20] that kinetic theory results are in agreement with Israel-Stewart's theory when gradients are assumed to be small. In 2+1 dimensions, shear stress tensor has 3 independent components. We have solved the relaxation equations for the 3 independent components and used the transversality condition ($\pi^{\mu\nu} u_\mu = 0$) and tracelessness condition ($\pi^\mu_\mu = 0$) to obtain the dependent components. The programme is more efficient and the transversality condition (and tracelessness condition) is maintained throughout the evolution. We have also checked that contribution of the term $[u^\mu \pi^{\nu\lambda} + u^\nu \pi^{\mu\lambda}] Du_\lambda$ is negligible in minimally viscous hydrodynamics [1].
 - [22] S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. C **71**, 034908 (2005) [Erratum-ibid. C **71**, 049901 (2005)] [arXiv:nucl-ex/0409015].
 - [23] U. W. Heinz, arXiv:hep-ph/0407360.
 - [24] T. Hirano and K. Tsuda, Phys. Rev. C **66**, 054905 (2002) [arXiv:nucl-th/0205043].
 - [25] D. Krieg and M. Bleicher, arXiv:0708.3015 [nucl-th].
 - [26] D. Molnar, arXiv:0707.1251 [nucl-th].